IT575 - Computational Shape Modeling
Assignment 3 - Laplace Beltrami operator

MATLAB codes for the LBO are provided on the course website, one using the weak formulation, while other using the Divergence theorem. Moreover, a few meshes have also been provided; demonstrate your solutions for the following questions on at least one mesh.

1. Implement the heat equation using either explicit or implicit discretization of the heat equation. Show sample plots of the solution, with heat distribution being randomly initialized.

2. Instead of using a function u on the mesh, what happens if the heat solution is computed for the coordinate functions x, y and z, i.e, $\frac{\partial x}{\partial t} = \Delta x$, and similarly for y and z. Plot a few meshes thus obtained.

3. Let us formulate the problem of denoising/smoothing of functions as a cost minimization process. Let f be the given noisy function on mesh M with LBO L (assume L is an approximation to $-\Delta$). The cost function is $C(g) = ||f - g||^2 + \lambda g^t L g$. The denoised signal g^* is defined as $g^* = \arg\min C(g)$. The first term in the cost function, the *data term* prevents the denoised signal g^* from being too different from f, while the second term, the *prior/regularizer* term promotes signal smoothness. Find a gradient descent procedure for minimizing C, implement it and show a few intermediate plots of this process, assuming a (a) random initialized function on a mesh, and (b) coordinate functions x , y and z. Justify that the second term promotes smoothness.

4. Let M be the mesh vertex coordinate array of a given mesh, while M_i be the mesh vertex coordinates of the mesh obtained by preserving i percentage of LBO eigenfunction coefficients corresponding to mesh M. Find out the percentage of coefficients to be preserved so that the error: $e = \frac{||M - M_i||}{||M||}$ where $|| \cdot ||$ is the usual $\mathbb{R}^3$ norm, is below 0.05. Show a few plots for M_i, and plots for a few eigenfunctions of the mesh M.

Submission details

1. Submit all MATLAB files and the report as a single `.zip` file on the course webpage, no later than 18:00 hrs, Friday 10 March 2017.