IT575 Computational Shape Modeling
Assignment 2 - Differential Geometry of Surfaces

1. Let $\sigma : W \subset \mathbb{R}^2 \to U \subset \mathbb{R}^3$ be a surface patch for any given regular embedded surface. Let $p = \sigma(u, v) \in S$ be a point in S. The patch σ can be reparameterized using the map $\Phi(u, v) = (\tilde{u}, \tilde{v}) = (f(u, v), g(u, v))$, where $\Phi : W \to \tilde{W}$ is a diffeomorphism, such that $\tilde{\sigma}(\tilde{u}, \tilde{v}) = \sigma(u, v)$. Figure out the relation between the first fundamental form at p in terms of σ and $\tilde{\sigma}$.

2. Let $\sigma(u, v) = (u, v, f(u, v))$ be a parameterization of a surface for $(u, v) \in \mathbb{R}^2$, where f is a degree n polynomial in the two variables u and v:

$$f(u, v) = a_0 + a_1 u + a_2 v + a_3 u^2 + a_4 uv + a_5 v^2 + \ldots + a_k u^n + a_{k+1} u^{n-1} v + \ldots + a_m v^n. \quad (1)$$

   The function f can be represented by the coefficient vector $a = (a_0, a_1, \ldots, a_m)$[1].
   Write a MATLAB function `mypolysurface.m` that takes an input vector a (of any length), a vector (u_0, v_0) specifying parameter values, and

   (a) plots the surface $(u, v, f(u, v))$ for $u \in [u_0 - 1, u_0 + 1], v \in [v_0 - 1, v_0 + 1]$ as an appropriately sampled mesh (use `mesh.m` or `surf.m`),

   (b) plots the basis of the tangent space[2] σ_u, σ_v at the point $\sigma(u_0, v_0)$, and the unit normal, (use `quiver3`),

   (c) plots the principal curvature directions scaled by the principal curvatures at $\sigma(u_0, v_0)$,

   (d) and outputs the Gaussian and Mean curvature at $\sigma(u_0, v_0)$ on the screen.

3. Given a surface patch $\sigma(u, v) = (u, v, f(u, v))$ where $f : \mathbb{R}^2 \to \mathbb{R}$ is an arbitrary smooth function, compute the

   (a) First fundamental form and unit normal to the surface at a point $\sigma(u, v)$,

   (b) The Second fundamental form and principal curvatures κ_1, κ_2 at a point $\sigma(u, v)$.

   (c) Plot examples of surfaces (using Question 2) with points where:

      i. $\kappa_1 > 0, \kappa_2 > 0$.

      ii. $\kappa_1 < 0, \kappa_2 > 0$.

      iii. $\kappa_1 > 0, \kappa_2 = 0$.

      iv. $\kappa_1 = \kappa_2 = 0$, but the surface is not a flat plane.

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[1] You can assume that the coefficient vector entries are contiguous and in the same order as given in Equation (1), with all other entries being zero.

[2] given a polynomial coefficient vector, it is not too difficult to analytically write its derivatives in terms of the coefficient vector; the derivatives of a polynomial will also be polynomials.

Submission Instructions

1. Write your answers as a LaTeX report; submit the generated pdf - `YourID_report_asg2.pdf`) along with `mypolysurface.m` as a single zip file - `YourID_asg2.zip` on the course webpage, **no later than Saturday** 4^{th} **February 2017, 8:00 am**.