

## SC 216

## Calculus with Complex Variables

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

Version 3 (Spring 2008)

## INSTRUCTIONS:

- There are 3 pages (6 pages double side). Ensure that you have all the pages.
- Answer **all questions**, writing clearly in the space provided.
- Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- Write your name and student number **clearly** at the top of each page.
- You have **two hours** to complete the test
- Marks for each question are indicated in brackets at right. You may use point form for your answers, but make sure the points are clear and unambiguous.

## FOR MARKER'S USE ONLY

Question	Possible	Received
1	5	
2	5	
3	5	
4	5	
5	5	
TOTAL	25	

## 1. ODE

- (a) Construct an example of an ordinary differential equation which has no solution and another example which has only one solution. (5)

2. IF

(a) Suppose we want to develop a method for finding Integrating Factor of a differential equation

$$M(x, y)dx + N(x, y)dy = 0$$

What we should do ? Sketch the method

(5)

3. l.i. / l.d.

- (a) Are the following statements true or false? If the statement is true, justify it by a short proof and if it is false give a counter example showing it is false.
- (b) If  $\phi_1, \phi_2$  are linearly independent functions on an interval  $I$ , they are linearly independent on any interval  $J$  contained inside  $I$ . (2.5)

- (c) If  $\phi_1, \phi_2$  are linearly dependent functions on an interval  $I$ , they are linearly dependent on any interval  $J$  contained inside  $I$ . (2.5)

4. (a) Find the radius of the convergence of the following series

$$F(a, b, c, x) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} x^n$$

Also show that  $F(a, b, c, x)$  is a solution of the equation

$$x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0.$$

(5)

## 5. Complex

(a) Show using method from complex analysis that

$$\int_0^{\infty} \frac{\cos mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}, m > 0.$$

(5)