

SC 105

Calculus and Complex Variables

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

Version 2 (Fall 2010)

INSTRUCTIONS:

- There are 3 double sided pages (5 printed pages). Ensure that you have all the pages.
- Answer **all questions**, writing clearly in the space provided.
- Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- Write your name and student number **clearly** at the top of each page before starting the exam.
- You have **one hour** to complete the test
- Marks for each question are indicated in brackets at right. You may use point form for your answers, but make sure the points are clear and unambiguous. I am more interested in your thought process.

FOR MARKER'S USE ONLY

Question	Possible	Received
1	5	
2	5	
3	5	
4	5	
TOTAL	20	

1. Basic

- (a) If the statement given below is **true** then **prove it** or if it is **false** then provide a **counter example**.

$$(z_1 z_2)^\alpha = (z_1)^\alpha (z_2)^\alpha \text{ for any non-zero complex numbers } z_1 \text{ and } z_2 \text{ and } \alpha (\neq 0) \in \mathbb{C}.$$

(5)

2. Analytic

- (a) Give an example of a function $f : \mathbb{C} \rightarrow \mathbb{C}$ which is non-analytic at any point of \mathbb{C} . Justify your answer. (5)

3. Singularity and Integration

- (a) Discuss the singularity of the function $f(z) = \ln z$ and find the integral $\int_C z^n \operatorname{Ln} z \, dz$ where C is the circle of radius 1 i.e, $|z| = 1$ in the anti-clockwise direction and n is an even integer. (5)

4. Simple problem

- (a) Without evaluating the integral show that $\left| \int_C \frac{dz}{z^2-1} \right| \leq \frac{\pi}{3}$, where C is the arc of the circle $|z| = 2$ from 2 to $2i$. (5)