

SC105- Calculus and Complex Variables

Practice Problems

Week: August 31, 2015

Tutorial Discussion Week: None

Tutorial Submission: None

Tutors: Krishna Gopal and Dixita Limbachiya

- (1) Prove that if $a_n \leq b_n \leq c_n$ and $\lim a_n = \lim c_n = L$ then $\lim b_n = L$ as $n \rightarrow \infty$.
- (2) Prove or give a counter example. Let a_n be a sequence such that $a_{n+1} - a_n \rightarrow 0$. Does a_n have to converge?
- (3) Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ which is continuous except at the integers.
- (4) Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ which is continuous only at 0.
- (5) Give an example of a function which is continuous only at the integers. Plot the graph.
- (6) Give an example of two functions, both discontinuous at 0, whose sum is continuous at 0.
- (7) Give an example of two functions, both discontinuous at 0. Whose product is continuous at 0.
- (8) Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ which is continuous at all irrationals points and discontinuous at all rational points of \mathbb{R} .
- (9) Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ which is discontinuous at all irrationals points and continuous at all rational points of \mathbb{R} .