SC 105

Calculus and Complex Variables

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

Version 1 (Fall 2009)

INSTRUCTIONS:

- There are 3 double sided pages (5 printed pages). Ensure that you have all the pages.
- Answer **all questions**, writing clearly in the space provided.
- Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- Write your name and student number **clearly** at the top of each page.
- You have **one hour** to complete the test
- Marks for each question are indicated in brackets at right. You may use point form for your answers, but make sure the points are clear and unambiguous. I am more interested in your thought process.

FOR MARKER'S USE ONLY

Question	Possible	Received
1	5	
2	5	
3	5	
4	10	
TOTAL	25	

1. Limit

   (a) Show by $\epsilon - N$ method that the following limit exist.

$$\text{Limit}_{n \longrightarrow \infty} \left\{ \frac{\sin n}{n} \right\} = 0. \tag{5}$$

2. Continuity and Differentiability

   (a) Give an example of a function which is continuos exactly at 1 point. Justify your answer. (5)

3. Taylor's Series

   (a) Obtain the Taylor's series expansion of the function $f(x) = \sin(x)$ about the point $x = \frac{\pi}{4}$ using Taylor's inequality. (5)

4. Several Variables

(a) Let $f(x, y)$ and $g(x, y)$ be two functions such that $f(0, 0) = 0 = g(0, 0)$ and for non-zero (x, y) they are defined below. Discuss their continuity at $(0, 0)$ and find out if their partial derivative f_x, f_y exists or not at $(0, 0)$.

$$f(x, y) = \frac{x^2+y^2}{|x|+|y|} \text{ and } g(x, y) = \frac{xy}{x^2+2y^2} \text{ for } (x, y) \neq (0, 0). \tag{10}$$