

SC 105

Calculus and Complex Variables

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

Version 1 (Fall 2010)

INSTRUCTIONS:

- There are 3 double sided pages (5 printed pages). Ensure that you have all the pages.
- Answer **all questions**, writing clearly in the space provided.
- Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- Write your name and student number **clearly** at the top of each page before starting the exam.
- You have **one hour** to complete the test
- Marks for each question are indicated in brackets at right. You may use point form for your answers, but make sure the points are clear and unambiguous. I am more interested in your thought process.

FOR MARKER'S USE ONLY

Question	Possible	Received
1	5	
2	5	
3	5	
4	10	
TOTAL	25	

1. Limit

- (a) Write down the $\epsilon - \delta$ definitions of right-hand and left-hand limits for a function $f : \mathbb{R} \rightarrow \mathbb{R}$. Using the appropriate definition show that

$$\lim_{x \rightarrow 0^+} \sqrt{x} = 0. \tag{5}$$

2. Continuity

- (a) Give an example of a function which is discontinuous everywhere on \mathbb{R} . Justify your answer. (5)

3. Differentiability

(a) Suppose that $f(x)$ is differentiable for all values of x such that

$$f(a) = a, f(-a) = -a \text{ and } |f'(x)| \leq 1, \text{ for all } x$$

Show that $f(0) = 0$.

(5)

4. Several Variables

- (a) Suppose that Arun and Anish in *SC105* class define two new distances d_1 and d_2 between two points in \mathbb{R}^2 as

$$d_1((x, y), (a, b)) = |x - a| + |y - b| \text{ and } d_2((x, y), (a, b)) = \max\{|x - a|, |y - b|\}.$$

They wanted to have their own definition. Can you help them in writing the ϵ and δ definitions of limit and continuity for functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ with respect to both the distances? Justify your answer and check whether they are on the right path or not? (10)