A primitive lab module for DIP*

November 13, 2006

Guide Lines

This lab is designed to motivate students to explore the vast field of image processing. It is an unsupervised learning module. Although it is termed so, it will be conducted strictly in a manner a tutorial is conducted, a kind of challenge response model. Few instructions :

- Student will have to work in a group of not more than three[1].
- Depending on the strength of the class a minimum number of group will be decided. A challenge[2] will be made available to the whole class if this minimum number of groups are ready to accept it.
- Each group is required to prepare a short report on their explorations.
- A challenge expires after fifteen days of its announcement. Submitting results and reports after fifteen days is not permissible.
- During the exploration student can use any reference, any software, any programming language. Student can even use the codes already available on the internet. The only constraint is that the student must acknowledge the source and the author, and should clearly state and mention the reference or links in the short report.
- It is possible to accept more than one challenge at a time. A total of five challenges need to be taken up during the course of study.
- Evaluation will be done in an extra session. A(All) group(s) will be asked to present their results of the challenge.

*A thought by Pratik Shah, Doctoral student, DAIICT

[1] Because more than three is a crowd

[2] A challenge will be in the form of an interesting set of problems.

Domain and scope

Think before you are ready to take a challenge. Here a short content of every challenge is given. The problems inside the challenge will require these concepts to be well understood by the student.

1. *Noise generation and analysis*
   Objective of this challenge is to understand the importance of noise and various sources of noise in image processing domain. Analysis part involves concepts like independency, correlatedness, whiteness, measure of noise etc. It involves fourier analysis and various measures like signal to noise ration. Challenge also talks about histogram as a tool for unfolding noise distribution. Many a times it is required to generate noise of given distribution with given characteristics, this challenge basically tries to give feel of such problems.

2. *Affine and projective transformations*
   Objective of this challenge is to get familiarity with the matrix algebra. When it is required to apply rotation, translation, scaling in image processing what kind of problems come up and how to deal with such problems is the central theme of this challenge. Geometry plays an important role in all the fields of science, and that is what makes image processing more interesting.

3. *Intensity transformations and filters*
   Concepts: histogram equalization, histogram specification, edge detectors, ideal and practical filters, order statistic filters, adaptive filters.

4. *Compression, hiding, codeing*
   Objective of this challege is to gain familiarity with various transform domain techniques, new data representations for binary images in terms of curves and surfaces, and codeing. It will also contain simulations of digital watermarks and the various attacks carried out on them.

5. *Morphology*
   Objective of this challenge to explore and establish connections between traditional linear system theory and the set theoretic morphology from an image processing perspective.

Challenge 1

1. Generate following images:

$$I_1(n_x, n_y) = Q(sin(2\pi f_x n_x T))$$
$$I_2(n_x, n_y) = Q(sin(2\pi f_x n_x T + 2\pi f_y n_y T))$$
$$I_3(n_x, n_y) = Q(A + cos(2\pi T(f_x n_x + f_y n_y)))$$

   where $n_x, n_y \in [0, 99]$, $f_x, f_y \in \{1, 10, 20\}$. $Q(f(n_x, n_y))$ quantizes the values of a real function to integers$\in [0, 255]$. T is a sampling time period. Take the sampling requency to be 10 times $\{f_x, f_y\}_{max}$. Find out the DFT for all the images generated. Interprete your results. Now change the sampling frequency from 10 times to 2 times $\{f_x, f_y\}_{max}$ and finally just the $\{f_x, f_y\}_{max}$. Find the DFT of the same. Explain the aliasing effect. Comment on the symmetry of fourier transform in 2-D.

2. Generate a checker board image with size 100×100 and 256 gray levels. Use only 0 and 255 as alternative block colours. Vary the block size from 2^2, 5^2,10^2 and 50^2 pixels. Find the DFT for each of the image. Comment on the result.

   Take a two dimensional step function defined on a discrete grid as;

$$I(x, y) = \begin{cases} 255 & \text{if } x \geq N_1 \text{ and } y \geq N_2 \\ 0 & \text{elsewhere} \end{cases}$$

   Let the discrete grid be finite and size be 50×50. Arbitrarily select N_1 and N_2 between $[1, 50]$. Find the DFT of image $I(x, y)$ and compare it with the one dimensional case.

   Consider the image given below;

$$I(x, y) = \begin{cases} 255 & \text{if } x \geq y,\, x, y \in [100, 100] \\ 0 & \text{elsewhere} \end{cases}$$

   Find the directional derivative of the image in the direction normal to the boundary of the two regions in the image. Find DFT for the original and the differentiated images. Comment on results.

3. Due to production fault the sensor of a camera is producing erroneous images of a captured scene. The sensor array is affected in a way such that there are total of 10% of the pixels in error. The probability of erroneous pixel being in the right half of the image is 80% and left part 20%. The error is due to saturation of values, which means erroneous pixel can be either 0 or 255 with equal probability. Take any image of size 100×100, with $[0, 255]$ gray-levels, make it look like asif it is captured by the faulty camera.

   Plot the histogram of both the original and the generated noisy image. Find DFT for both no-noisy and noisy image. Write down your findings.

4. Generate additive zero mean gaussian noise, add it up with sample image with different variances for noise. Calculate SNR, PSNR and plot the histogram for each. Plot PDF and CDF of generated noise. Explain what is PSD? What is whiteness in connection with noise? What different kinds of noise have you encountered in image processnig?

5. You must have observed by now that sometimes while calculating DFT for images the images are premultiplied by $(-1)^{x+y}$, where x and y are image coordinates, explain why?

Challenge 2

1. Does every intersecting lines in $\mathbf{R}^2$ intersect in our discrete grid? Consider a line in an image as a subset of $\mathbf{N}^2$ and intersection of two such lines as conventional set intersection. Now create an image with two lines such that they do not intersect by the way we have described the intersection point above. Have you heard about diophantine equations? Can you establish some connections here? While working with functions defined on real line we require knowledge of the real number system and its various structures and properties(what we study in analysis!), if we consider images to be functions defined on $\mathbf{N}^2$, we must understand the nature and structure of it. Rosenfeld's work on digital topology may help in understanding so[3].

2. Take a solid object of any shape[4]. Task is to rotate this object with resepect to its center of gravity by 45 degrees. After rotation store this result as an image. What kind of difficulties did you face while storing it as an image? What do you think after this exercise - is rotation quite simple?

3. Take an image of size $N \times N$, decimate it by factor of two. Now using various interpolation schemes like bilinear, splines and higher order polynomials interpolate $N/2 \times N/2$ image to the original size and observe the results, comment on them. Increase the decimation factor to four and eight. Try to reconstruct the original one. What do you observe? Are you aware of super-resolution?

4. Take two solid ellipse(s). First of all find out how many different configurations are there for these ellipse(s) which can be used for gear mechanism. Have you heard about elliptical gears[5]? Generate animated sequence of images for one full rotation of elliptical gears.

5. How to show motion in a single image? Easy for those who read comics or who love sketching. Just show motion by action lines. Take or create any simple animated sequence in which atleast one object is in motion. Now for that object according to animated motion try to generate a single image with action lines[6].

6. Finally if you still have time left you can create an animation showing a man who can jump and run and bla bla bla[7].

[3]A. Rosenfeld (1979) Digital topology. American mathematical Monthly 86:621-630

[4]Whenever this kind of statements are made here it strictly means take a closed curve fill it up with some colour or graylevel or just use binary scheme and create an image of it. Remember we are working with images.

[5]For motivation of this exercise just browse through the literature and see what are the different applications of elliptical gear.

[6]One such reference is, Cartoon Blur: Non-Photorealistic Motion Blur by yuya Kawagishi, Kazuhide Hatsuyama, Kunio Kondo(CGI03)

[7]This problem will not be considered in grading, it is just a fun problem.

Challenge3

It is always possible to draw a straight line
passing through any three points in a plane.
- *Anonymous*[8]

1. In laplacian operator why the sum of all elements of the kernel becomes zero? It is said to be the approximation of second order derivative, write down the construction of discrete laplacian operator from the continuous second order derivative. What difference does it make if the sum is not zero? For a laplacian operator find out the corresponding frequency domain representation. Take an image apply laplacian operator in spatial as well as frequency domain. Compare the results.

2. Take an image and repetitively pass it through a gaussian filter with mean zero and some fixed variance. In other words if $I(x, y)$ is an image and $G(\cdot)$ is a gaussian filter, than, applying gaussian filter n times means; $G^n(I(x, y)) = G(G(G(...G(I(x, y)))))$. What happens as $n \to \infty$? Do this process in fourier domain also. Compare the results.

3. In the first challenge we dealt with the problem of modeling the faulty image capturing sensor. Now suppose we captured the image with such device and end up with a noisy image. What filters are used to deal with such kind of noise? Have you heard of Median and Adaptive median filters? Try to filter out the salt-pepper noise in the image. "Median filter is a linear filter." Comment, correct and justify the sentence.

   Give an example where you would use adaptive median filter. In adaptive median filter, what is the criteria for increasing and decreasing the kernel size, is it in any sense dependent on noise distribution?

4. Are Gaussian and Laplacian operators commutative? Take an image apply first gaussian and than laplacian and for the same image first apply laplacian and than gaussian, compare the results. Give mathematical proof of your conclusion.

5. Prove that there exists a function for which the fourier transform of the function is the function itself. Find out this function.

6. Take a low contrast image. Try varying the histogram of the image by transforming the intensity values. Have you ever used GIMP? There is an option for varying the histogram, look into it. What is histogram equalization? Apply histogram equalization to the image. What happens if the equalized image is again equlized? Prove that histogram equalization is an idempotent operation.

[8] Consider the pencil to be thick enough.

Challenge 4

The junior Bat asked the senour Bat
A question most profound:
'How do the humans down below
Hang by their feet from the ground?'
(Dick Smithells and Ian Pillinger: Alphabet Zoop)

1. Implement erosion and dilation as building blocks of a binary morphological system. Take an image apply opening, closing and comment on the results. What is convexity? For a symmetric convex structing element apply erosions recursively on the image and observe the result. Do the same for dilation. What do you observe?

2. Implement various applications(atleast three) for binary images like skeletoning, thinning, boundary detection, hit or miss operation, etc.

3. Have you heard about PECSTRUM(Pattern Spectrum)? What is it? For one image with different size of squares in it, try to find out the PECSTRUM of this image. Explain results.

4. What is Minkowski's sum? Represent all the morphological operations using Minkowski's sum. Say something about convolution, correlation and minkowski's sum.

5. Implement atleast basic erosion and dilation for grayscale morphology. Are morphological operators linear? Give some analogies between traditional linear processing systems and morphology.

6. Is there any way to represent signals/images as collection of sets? Read about unification of linear and nonlinear signal shaping techniques. (Work done by P.Maragos, W.Schafer, G.Matheron) Write few lines about what you found.

Challenge 5

Everyone knows what a curve is, until he has studied enough mathematics
to become confused through the countless number of possible exceptions. [9]

1. How would you store a given binary curve? Give examples for your schemes. What if there are more than one curves in an image? Are you aware of some sort of data structure supporting these kinds of representations? Given two chain codes of a curve what can you say about them by only looking at their chain codes? Is there any way to find out the shape of the region covered by the curve which is chain coded? Implement a scheme which from a given chain code representation classifies the curve as circle, square and triangle. [10]

2. What is a generalized inverse of a given matrix? Have you heard about singular value decomposition, cholesky decomposition? Explaing them with the help of examples and also in terms of vector spaces and subspaces.

3. For a given image find out Fourier, DCT, KL, Wavelet transforms. First take a small matrix and apply all transforms on it, analyse and write your observations. Write down the theory behind each transform. Which is the best and why?

4. Implement a simple DCT based compression scheme. What are the parameters to be taken care of while compressing an image? How is the quality of a compressed image measured? What is arithmetic coding?

5. There are lot of demonstrations available on internet regarding watermarking schemes. One of them is DCT based scheme. Pick any one of the demo, understand it carefully and give answers to these questions in form of explanation as to what happens to the watermark if

   (a) I rotate the watermarked image by 30 degrees

   (b) I apply JPEG compression on watermarked image

   (c) I crop the watermarked image

   (d) I equalize the watermark image

6. Suppose you have used some watermarking scheme to watermark an image. Now if some one takes a print out of that image and scans it and stores it as another image. Will you be able to get back the watermar from this print-scan image? Propose a scheme to deal with such situation.

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[9] *Quoted in C B Boyer, The Invention of Analytic Geometry, Scientific American 180 (1949)*

[10] www.citr.auckland.ac.nz/∼rklette/Books/MK2004/pdf-LectureNotes/04slides.pdf
Encoding:ieeexplore.ieee.org/iel5/8159/23997/01096361.pdf
A Novel Direction Chain code-based Image Retrieval: ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1357195