

SC116- Algebraic Structures

Home Work 12

Week: November 5, 2013

Tutorial Discussion Week: November 18, 2013

- (1) Identify which algebraic structure shown below forms a ring under the given operation of addition and multiplication. If it is a ring state whether it is commutative, whether it has unity and whether it is a field. If not a ring tell us why it is not a ring. (a) $S = \mathbb{Z}^+$, with usual addition and multiplication (b) $S = \mathbb{Z} \times \mathbb{Z}$ with addition and multiplication by components (c) $S = 2\mathbb{Z} \times \mathbb{Z}$ with addition and multiplication by components (d) $S = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ with usual addition and multiplication (e) $S = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ with usual addition and multiplication
- (2) Describe all the units in the given ring.
(a) \mathbb{Z} (b) $\mathbb{Z} \times \mathbb{Z}$ (c) \mathbb{Q} (d) $M_2(\mathbb{Z}_2)$: Set of all 2×2 matrices over \mathbb{Z}_2 . (e) \mathbb{Z}_{p^s} , where p is a prime and s a positive number (f) $\mathbb{Z}[i] = \{a + ib \mid a, b \in \mathbb{Z}\}$.
- (3) Find all the solutions of the equation $x^2 + x - 6 = 0$ over \mathbb{Z}_{14} .
- (4) Find the characteristic of the given ring.
(a) $2\mathbb{Z}$ (b) $\mathbb{Z} \times \mathbb{Z}$ (c) $\mathbb{Z}_3 \times \mathbb{Z}_3$ (d) $\mathbb{Z}_3 \times \mathbb{Z}_4$ (e) \mathbb{Z}_{p^s} , where p is a prime and s a positive number.
- (5) Describe the field of quotients of the integral domain $\mathbb{Z}[i] = \{a + ib \mid a, b \in \mathbb{Z}\}$ in \mathbb{C} .
- (6) Find the factorization of $x^4 + 4$ into linear factors in $\mathbb{Z}_5[x]$.
- (7) Find all ideals N of \mathbb{Z}_{12} . In each case, compute the factor ring \mathbb{Z}_{12}/N .
- (8) Find all maximal ideals of \mathbb{Z}_6 .
- (9) Find all maximal ideals of $\mathbb{Z}_2 \times \mathbb{Z}_4$.
- (10) Find all $c \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x]/(x^2 + c)$ is a field.
- (11) Find all $c \in \mathbb{Z}_5$ such that $\mathbb{Z}_5[x]/(x^2 + x + c)$ is a field.