

SC116- Algebraic Structures

Home Work 10

Week: October 14, 2013

Tutorial Discussion Week: October 21, 2013

- (1) Are the following mappings automorphisms of their respective groups? (a) G group of integers under addition, $T : x \rightarrow -x$. (b) G group of positive reals under multiplication, $T : x \rightarrow x^2$. (c) G cyclic group of order 12, $T : x \rightarrow x^3$. (d) G is the group S_3 , $T : x \rightarrow x^{-1}$.
- (2) Let G be a group of order 4, $G = \{e, a, b, ab\}$, $a^2 = b^2 = e$, $ab = ba$. Determine $Aut(G)$.
- (3) Let G be a group of order 9 generated by elements a, b , where $a^3 = b^3 = e$. Find all the automorphisms of G .
- (4) For $G = S_3$, show that G is isomorphic to $Inn(G)$.
- (5) For any group G , show that $Inn(G)$ is a normal subgroup of $Aut(G)$.
- (6) Construct a non-abelian group of order 55.
- (7) Show that any group of order p^2 , where p is a prime number, must have a normal subgroup of order p .
- (8) Show that any group of order p^2 , where p is a prime number, is abelian.
- (9) Show that any group of order $2p$, where p is a prime number, must have a subgroup of order p and that this subgroup is normal in G .
- (10) If $o(G) = pq$, where p and q are distinct prime numbers and if G has a normal subgroup of order p and a normal subgroup of order q , show that G is cyclic.
- (11) Find the orbits and cycles of the following permutations:
(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$. (b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$.
- (12) Write the permutations in Q11 as a product of disjoint cycles.