SC116- Algebraic Structures

Home Work 4

Week: August 12, 2013

Tutorial Discussion Week: August 19, 2013

(1) Are the maps $T : V \to W$ given below, linear transformations?
(a) Let $V = \mathbb{R}^2$ and $W = \mathbb{R}^3$ with $T(x, y) = (x + y + 1, 2x - y, x + 3y)$. (b) Let $V = W = \mathbb{R}^2$ with $T(x, y) = (x - y, x^2 - y^2)$. (c) Let $V = W = \mathbb{R}^2$ with $T(x, y) = (x - y, |x|)$. (d) Let $V = W = \mathbb{R}^4$ with $T(x, y, z, w) = (z, x, w, y)$.
(2) Does there exist a linear transformation T described below?
(a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, -1, 1) = (1, 2)$ and $T(-1, 1, 2) = (1, 0)$? (b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, 0, 1) = (1, 2), T(0, 1, 1) = (1, 0)$ and $T(1, 1, 1) = (2, 3)$ (c) $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, 0, 1) = (1, 2), T(0, 1, 1) = (1, 0)$ and $T(1, 1, 2) = (2, 3)$.
(3) Define a map $T : \mathbb{C} \to \mathbb{C}$ by $T(z) = \bar{z}$, the complex conjugate of z. Is T a linear transformation over $\mathbb{C}(\mathbb{R})$?
(4) Show that there exists infinitely many linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, -1, 1) = (1, 2)$ and $T(-1, 1, 2) = (1, 0)$.
(5) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a function that reflects every point in $\mathbb{R}^3$ about the X-axis. Find its matrix with respect to standard ordered basis of $\mathbb{R}^3$.
(6) For each linear transformation given in Q1 above find its matrix of linear transformation with respect to standard ordered basis.
(7) For $P_n(\mathbb{R})$ = {set of all polynomials of degree at most n with real coefficients}, define a linear operator $\mathcal{D} : \mathcal{P}_n(\mathbb{R}) \to \mathcal{P}_n(\mathbb{R})$ by

$$\mathcal{D}(a_0 + a_1 x + a_2 x^2 + \dots a_n x^n) = a_1 + 2a_2 x + \dots + n a_n x^{n-1}.$$

Describe the null space $\mathcal{N}(\mathcal{D})$ and range space $\mathcal{R}(\mathcal{D})$.
(8) Find a linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ for which $\mathcal{R}(T) = LS((1, 2, 0), (0, 1, 1), (1, 3, 1))$?
(9) Show that the solutions to the linear differential equation $\frac{d^2u}{dt^2} = u$ form a vector space. Find two independent solutions to give a basis for the solution space.
(10) The space of all 2×2 matrices has the four basis "vectors"

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

For the linear transformation of *transposing*, find its matrix of transformation with respect to this basis.