SC 611 Mathematics for Computer Science

Solutions for Home Work6
week: Sep 30, 2011
Tutorial Discussion Week: Sep 30, 2011
Tutorial Submission Deadline: Sep 30, 2011

1. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?
   Sol: We assume that the row has a distinguished head. Consider the order in which the men appear relative to each other.
   There are n men, and all of the P (n; n) = n! arrangements is allowed.
   Similarly, there are n! arrangements in which the women can appear.
   Now the men and women must alternate, and there are the same number of men and women; therefore there are exactly two possibili-ties:

   (a) either the row starts with a man and ends with a woman (MW MW : : : MW)

   (b) or else it starts with a woman and ends with a man (W MW M : : : W M).

   arrange the men among themselves, arrange the women among themselves, and decide which sex starts the row.
   By the product rule there are $n!.n!.2 = 2(n!)^2$ ways in which this can be done.

2. How many permutations of the letters ABCDEFG contain

   (a) the string BCD
   Sol: Count the permutations of $A\$EFG$ where $ represents BCD : 5*4*3*2*1 = 120

   (b) the string CFGA
   Sol: Count the permuations of $B\&DE$ where & represents CFGA : 4*3*2*1 = 24

   (c) the string ABC and CDE
   Sol: Must contain ABCDE; count permutations of $\$FG$: 3! = 6

   (d) the string CBA and BED
   Sol: It is not possible for B to be in the specified positions.

3. A club has 25 members.

   (a) How many ways are there to choose four members of the club to serve on an executive committee?
   Sol: Order not important

   $$C(25, 4) = \frac{25!}{21!4!} = \frac{25 * 24 * 23 * 22}{4 * 3 * 2 * 1} = 25 * 23 * 22 = 12,650$$

   (b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club?
   sol: Order is important

   $$P(25, 4) = \frac{25!}{21!} = 303,600$$

4. **Sol:**

   (a) First, place the first person in the north-most chair:
   Only one possibility

   (b) Then place the other 5 people
   There are P(5,5) = 5! = 120 ways to do that By the product rule, we get 1*120 =120

   (c) Alternative means to answer this:
   There are P(6,6)=720 ways to seat the 6 people around the table For each seating, there are 6 rotations of the seating Thus, the final answer is 720/6 = 120

5. **Sol:**

   (a) Find the expansion of $(x+y)^6$ using Binomial Theorem?

$$(x+y)^6 = \binom{6}{0}x^6 + \binom{6}{1}x^{6-1}y^1 + \binom{6}{2}x^{6-2}y^2 + \binom{6}{3}x^{6-3}y^3 + \binom{6}{4}x^{6-4}y^4 + \binom{6}{5}x^{6-5}y^5 + \binom{6}{6}x^{6-6}y^6$$

$$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

   (b) Find the coefficient of x^5y^8 in $(x+y)^{13}$.

$$Coefficient\ of\ x^{n-r}y^r = \binom{n}{r}$$

   where n=13, r=8

$$Coefficient\ of\ x^5y^8 = \binom{13}{8} = 1,287$$

   (c) Give a formula for the coefficient of x^k in the expansion of $(x+\frac{1}{x})^{100}$, where k is an integer.
   We know that

$$Coefficient\ of\ x^{n-r}y^r = \binom{n}{r}\ where\ y = \frac{1}{x}$$

$$x^{n-r}.\frac{1}{x^r} = x^k$$

$$n - 2r = k$$

$$r = \frac{n-k}{2}$$

   :. Coefficient of x^k in the expansion of $(x+\frac{1}{x})^n$ is$\binom{n}{\frac{n-k}{2}}$
   :. Formula for the coefficient of x^k in the expansion of $(x+\frac{1}{x})^{100}$ is s$\binom{100}{\frac{100-k}{2}}$